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Harmonic Mean Labelling For Some Special Graphs

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Abstract :- A graph G=(V,E) with p vertices and q edges is said to be a mean graph if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 0,1,2,…..q in such a way that when each edge $e = uv$ is labelled with $\left[\frac{f(u)+f(v)}{2}\right]$ if $f(u)+f(v)$ is even and $\left[\frac{f(u)+f(v)+1}{2}\right]$ if $f(u)+f(v)$ is odd, then the resulting edge labels are distinct. f is called a mean labelling of G.

Here, we investigate the mean labelling of caterpillar, $C_n^{(2)}$ dragon, arbitrary super subdivision of a path and some graphs which are obtained from cycles and stars.

Keywords and phrases: *Mean graph, dragon, super subdivision of a graph, caterpillar*

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I. INTRODUCTION

By a graph we mean a finite undirected graph without loops or parallel edges. The vertex set is denoted by V(G) and the edge set is denoted by E(G). A cycle of length n is C_n and a path of length n is referred P_n. For all detailed survey of graph labelling, we refer to J.A.Gallian. For all other standard terminology and notations we follow Harary.

The concept of Mean labelling has been introduced by S.Somasundaram and R.Ponraj and also S.Somasundaram and S.S.Sandhya introduced Harmonic mean labelling. Motivated by the above works we introduced a new type of labelling called Harmonic mean labelling.

In this paper we investigate the subdivision of Harmonic mean labelling of graphs. We will provide brief summary of definitions and other information which are necessary for our present investigation.

In this paper, we consider only finite, simple and undirected graphs. Let $G(V,E)$ be a graph with p vertices and q edges. For notations and terminology we follow. In a graph G, the subdivision of an edge uv is the process of deleting the edge of G is subdivided exactly once, then the resultant graph is denoted by S(G) and is called the subdivision graph of G.

Somasundaram and Ponraj have introduced the concept of mean labelling of graphs. An assignment $f : V(G) \to \{0,1,2,....,q\}$ is called a mean labelling if whenever each edge e=uv is labelled with $\left[\frac{f(u)+f(v)}{2}\right]$ if $f(u)+f(v)$ is even and $\left[\frac{f(u)+f(v)+1}{2}\right]$ if $f(u)+f(v)$ is odd, then the resulting edge labels are all distinct. Any graph that admits a mean labelling is called a mean graph.

Many results on mean labelling have been proved. In a similar way, Somasundaram, Ponraj and Sandhya have introduced the concept of harmonic mean labelling of a graph. An assignment $f : V(G) \to \{1,2,.....,q+1\}$ is called a harmonic mean labelling if whenever each edge $e = uv$

Is labelled with $\left[\frac{2f(u)f(v)}{f(u)+f(v)}\right]$ or $\left[\frac{2f(u)f(v)}{f(u)+f(v)}\right]$ then the edge labels are distinct. Any graph that admits a harmonic mean labelling is called a harmonic mean graph.

More results on harmonic mean labelling have been proved. A well collection of results on graph labelling has been done in the survey.

In this paper, we establish the harmonic mean labelling of some standard graphs like subdivision of star $S(K_{1,n})$, subdivision of bistar $S(B_{n,m})$, the disconnected graphs $S(K_{1,n}) \cup kC_m$ etc.

This dissertation entitled **"HARMONIC MEAN LABELLING OF SUBDIVISIONAND RELATED GRAPHS"** consists of four chapters.

In chapter-1 some Basic definitions and its examples are described.

In chapter -2 Harmonic mean Labelling of Some Cycle Related graphs.

In chapter-3 Harmonic mean Labelling of subdivision and Related special graphs.

In chapter -4 Harmonic mean Labelling for some graphs

In chapter -5 k- Harmonic mean Labelling for some graphs and subdivision of contra harmonic mean graphs.

II. PRELIMINARYT DEFINITIONT

Definition 1.1:

A graph $G(V,E)$ with p vertices and q edges with it is possible to label the vertices x∈V with distinct lables from 1,2…q+1 in such a way that wheh edge e=uv is labeled with $f(e=uv)=\left[\frac{2f(u)f(v)}{f(u)f(v)}\right]$then the edge labels are distinct. Then, itis called a Harmonic mean graph ifIn this case, f is called a **Harmonic mean labelling** of G.

Definition 1.2:

The **duplication** of v is called a graph G(v) obtained from G by adding a new vertex v'with N(v′)=N(v). Here, v be a vertex of a graph G.

Definition 1.3:

Let e=uv be an edge of G. Then **duplication of an edge** e=uv is a graph G(uv) obtained from G by adding a new edge u′v′ such that N(u')=N(u)∪{v'}-{v} and N(v′)= N(v)∪{u'}-{u}

CHAPTER – II
HARMONIC MEAN LABELLING OF SOMECYCLE RELATED GRAPHS

Theorem 2.1:

The graph obtained by duplicating an arbitrary edge in cyclen is a Harmonic mean graph.

Proof:

Let $C_n= v_1v_2\ldots v_nv_1$ be the cycle.

Let $e'= u_1'u_2'$ be the duplicated edge of $e= u_1u_2$

Now we define f: $V(G(u_1u_2)) \rightarrow \{1,2\ldots q+1\}$ by

$f(u_1) = 1$

$f(u_2) = 2$

$f(u_3) = 3$

$f(u_i) = i+1, 4 \le i \le n$

$f(u_1') = n$

$f(u_2') = n+1$

Hence f is a Harmonic mean labelling of duplicated graph $G(u_1u_2)$

III. HARMONICT MEANT LABELLINGT OFT SUBDIVISIONT ANDT RELATEDT GRAPHS

Theorem 3.1

The disconnected graph $(K_{1,n}) \cup kC_m$ is a harmonic mean graph for

$1 \le n \le 5$, $m \ge 3$ and $k \ge 0$

Proof

Let $V(S(K_{1,n}) \cup kC_m)=\{v;$ $u_1,u_2,\ldots,u_n;v_1,v_2,\ldots,v_n;w_{11},w_{12},\ldots,w_{1m};$ $w_{21},w_{22},\ldots,w_{2m};\ldots w_{k1},w_{k2},\ldots,w_{km}\}$ and $E(S(K_{1,n}) \cup kC_m)=\{vu_i,u_iv_i|1\le i\le n\}\cup[U_{i=1}^{k}((U_{j=1}^{M-1}\{W_{ij}\ \ W_{ij+1}\})\cup\{W_{im}W_{i1}\}]$.

Here $p=2n+km+1$ and $q=2n+km$.

Define a function f: $V(S(K_{1,n}) \cup kC_m) \rightarrow \{1,2,\ldots,q+1\}$ by

$f(v) = 2n+1;$

$f(u_i) = n+i, 1 \le i \le n;$

$f(v_i) = n+1-i, 1 \le i \le n$ and

$f(w_{ij}) = (2n+1)+(i-1)m+j, 1 \le i \le k, 1 \le j \le m.$

Then the induced edge labels of $(K_{1,})$ are given below:

$f^*(u_nv_n) = 1;$

$f^*(u_iv_i) = n+1+i, 1 \le i \le n;$

$f^*(u_iv_i) = n+2-i, 1 \le i \le n-1;$

and the set of all edge labels of kC_m is $\{2(n+1), 2n+3,\ldots,2n+km+1\}$.

Therefore the set of all edge labels of $(K_{1,}) \cup kC_m$ is $\{1, 3, 4 \ldots .2n+km+1\}$.

Hence $(K_{1,}) \cup kC_m$ is a harmonic mean graph for $1 \le n \le 5, \ge 0$ and $m \ge 3$.

Hence the theorem.

IV. HARMONIC MEANLABELLING FOR SOMESPECIAL GRAPHS

Theorem 4.1:

An (m, n) kite graph G is a Harmonic mean graph.

Proof:

Let $u_1\ u_2\ u_3\ldots\ldots u_mu_1$ be the given cycle of length m and $v_1v_2\ldots\ldots v_n$ be the given path of length n.

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Define a function $f : V(G) \rightarrow \{1,2\ldots.q+1\}$ by

$f(u_i) = i, \ 1 \leq i \leq m,$

$f(v_i) = n+i, \ 1 \leq i \leq n$

Then the edge labels of the cycle are

$f(u_1u_m) = 1,$

$f(u_iu_{i+1}) = i+1, \ 1 \leq i \leq m$ and

The edge labels of the path are $\{m+1, m+2\ldots. m+n\}$

Hence the kite graph is a Harmonic mean graph

V. CONCLUSION

In this thesis, we discuss Harmonic mean labelling behaviour of some cycle related graphs such as duplication, joint sum of the cycle and identification of cycle. Also we investigate Harmonic mean labelling behaviour of Alternate Triangular Snake A(Tn), Alternate Quadrilateral Snake A(Qn).

In this paper, we establish harmonic mean labels of some well known subdivision graphs and some disconnected graphs.In this paper we prove the Harmonic mean labelling behaviour for some special graphs.

In this paper, we establish the harmonic mean labelling of some standard graphs like subdivision of star $(K_{1,})$, subdivision of bistar $(B_{n,})$,the disconnected graph $S(K_{1,n}) \cup kC_m$.

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