Triangular Fuzzy Matrices

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Abstract: In this paper, some elementary operations on triangular fuzzy numbers are defined. we also define some operations on triangular fuzzy matrices such as trace and triangular fuzzy determinant. Using elementary operations. Some important properties of TFMS are presented. The concept of adjoints on TFM is discussed and some of their properties are. some special types of TFMs are defined and a number of properties of these TFMS are presented.

Keywords: *Triangular fuzzy numbers, triangular fuzzy number arithmetic, Triangular fuzzy matrices, Tringular fuzzy determinant. Circulant triangular fuzzy number matrices.*

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1. Introduction:

Mathematics deals with both quantitative relationships and such relationships are very conveniently represented by matrices. The applications of matrices are not only in the branches of mathematics but also in real world problems. Fuzzy mathematics has a very wide range of application in Engineering physical science in Linguistic and in numerous other areas.

A fuzzy set can be defined mathematical by assigning to each possible individual in the inverse of discourse value representating its grade of membership in the fuzzy set. Because full membership and full non membership in the fuzzy set can still be indicated by the values of 1 and 0 respectively we can consider the concept of a crisp set to be a restricted case of the more general concept of a fuzzy set for which only these two grades of memberships are allowed. The fuzziness can be represented by different ways one of the most useful representation is membership function. Also depending the nature and shape of the membership function the fuzzy number can be classified in different forms, such as traingular fuzzy number trapezoidal fuzzy number.

The fuzzy matrices introduced first time by Thomason and discussed about the convergence of powers of fuzzy matrix several authors presented a number of result on the convergence of power sequence of fuzzy matrices, Ragabetal presented some properties on determinant and adjoint of sequence fuzzy matrix. Ragabetal presented some properties of the min-max composition of fuzzy matrices kim presented some important results on determinant of a square fuzzy matrices.

2. Basic concept and Preliminaries :

Definition 2.1: A fuzzy set is characterized by a membership function mapping the elements of a domain, space or universe of discourse X to the unit interval [0, 1].

A fuzzy set A in a universe of disource X is defined as the following set of pair.

$A = \{(x, \mu_A (x)) : x \in X\}$

Here $\mu_A : X \to [0, 1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ is called the fuzzy set A.

Definition 2.2: A fuzzy set A of the universe of discourse X is called Normal fuzzy set implying that there exists at least one $x \in X$ such that $\mu_A (x) = 1$.

Definition 2.3A Traingular fuzzy matrix of order m x n is defined as $A = (a_{ij})_{mxn}$ where $a_{ij} = \langle m_{ij}, \alpha_{ij}, \beta_{ij}\rangle$is the ijth element of A, m_{ij} is the mean value of a_{ij} and α_{ij}, β_{ij} are the left and right spreads of a_{ij} respectively.

As for classical matrices the following operation on Triangular fuzzy matrices.

Let $A = (a_{ij})$ and $B = (b_{ij})$ be two Triangular fuzzy matrices of same order then we have the following.

i) $A + B = (a_{ij} + b_{ij})$

ii) $A - B = (a_{ij} - b_{ij})$

iii) For $A = (a_{ij})_{mxn}$ and $B = (b_{ij})_{nxp}$

$A . B = (c_{ij})_{mxp}$ where $c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$

$i = 1, 2 \ldots.. m$ and $j = 1, 2 \ldots\ldots\ldots. p$

iv) $A^{k+1} = A^k A^1$

v) $A' = (a_{ij})$ (the transpose of A)

vi) $k . A = (ka_{ij})$ where k is a scalar.

Definition 2.4: A Triangular fuzzy matrices is said to be a pure null Triangular fuzzy matrices if all its entries are zero i.e., all elements are ⟨0, 0, 0⟩ this matrix is denoted by O.

Definition 2.5:A Triangular fuzzy matrices is said to be a fuzzy null Triangular fuzzy matrices if all elements are of the from $a_{ij} = \langle 0, \in_1, \in_2\rangle$, where $\in_1 . \in_2 \neq 0$.

Definition 2.6:A square Triangular fuzzy matrices is said to be a fuzzy unit Triangular fuzzy matrices if$a_{ii} = \langle 1, \in_1, \in_2\rangle$and $a_{ij} = \langle 1, \in_3, \in_4\rangle$ for i ≠ j for all i, j where $\in_1 . \in_2 \neq \in_3 . \in_4 \neq 0$.

Definition 2.7:A square Triangular fuzzy matrices is said to be a pure unit Triangular fuzzy matrices if $a_{ii} = \langle 1, 0, 0\rangle$ and $a_{ij} = \langle 0, 0, 0\rangle$, i ≠ j for all i, j It is denoted by I.

Definition 2.8:A square Triangular fuzzy matrices A = (a_{ij}) is said to be a Fuzzy TFM if either $a_{ij} = \langle 0, \in_1, \in_2\rangle$ for all i > j or $a_{ij} = \langle 0, \in_1, \in_2\rangle$ for all i <j ; i, j = 1, 2 …. n and $\in_1 . \in_2 \neq 0$.

Definition 2.9:A square Triangular fuzzy matrices A = (a_{ij}) is said to be symmetric Triangular fuzzy matrices if $A = A^1$ i.e., if $a_{ij} = a_{ji}$ for all i, j.

Definition :2.10A matrix A = (a_{ij}) is called a fuzzy matrix, if each element of A is a fuzzy number we may represent nxn fuzzy matrix $A = (a_{ij})_{nxn}$ such that $a_{ij} = \langle a_{ij}, b_{ij}, c_{ij}\rangle$.

3.TRIANGULAR FUZZY NUMBER

A triangular fuzzy number denoted by M = ⟨m, α, β⟩ has the membership function

$$\mu_M(x) = \begin{cases} 0 & \text{for } x \le m - \alpha \\ 1 - \frac{m-x}{\alpha} & \text{for } m-\alpha < x < m, \alpha > 0 \\ 1 & \text{for } x = m \\ 1 - \frac{x-m}{\beta} & \text{for } m < x < m+\beta, \beta > 0 \\ 0 & \text{for } x \ge m+\beta \end{cases}$$

TRACE OF TFM

Trace of TFM. The trace of a square TFM A = (a_{ij}) denoted by tr(A) is the sum of the principal diagonal elements. In other words tr(A) = $\sum_{i=1}^{n} a_{ii}$

Theorem 3.1: The product of two pure lower triangular TFMS of order nxn is also a pure lower triangular TFM.

Proof:

Let A= (a_{ij}) and B= (b_{ij}) be two lower triangular TFMS.

Where $a_{ij} = \langle m_{ij}, \alpha_{ij}, \beta_{ij}\rangle$ and $b_{ij} = \langle n_{ij}, \gamma_{ij}\, \delta_{ij}\rangle$

Since A and B are pure lower triangular TFMS then a_{ij} = ⟨0,0,0⟩ and b_{ij} = ⟨0,0,0⟩ for all $i<j; i,j = 1,2 \ldots\ldots\ldots\ldots n$ s

Let A.B = C = (c_{ij}) where $c_{ij} = \sum_{k=1}^{n} a_{ik} \cdot b_{kj}\, b_{kj}$

$$= \sum_{k=1}^{n} \langle m_{ik}, \alpha_{ik}, \beta_{ik}\rangle \cdot \langle n_{kj}, \gamma_{kj}, \delta_{kj}\rangle$$

Now, show that c_{ij} = ⟨0,0,0⟩ if i<j

For i,j = 1,2……………….n

For i>j, a_{ik} = ⟨0,0,0⟩ and

For k= 1,2…………….. i+1

Similarly b_{kj}= ⟨0,0,0⟩for k= i, i-1…………n.

Therefore $c_{ij} = \sum_{k=1}^{n} a_{ik}\, b_{kj}$

$$= \sum_{k=1}^{i+1} a_{ik}\, \beta_{kj} + \sum_{k=1}^{n} a_{ik} \cdot b_{kj}$$

= ⟨0,0,0⟩

Now $c_{ii} = \sum_{k=1}^{n} a_{ik} \cdot b_{ki}$

$$= \sum_{k=1}^{i+1} a_{ik} \cdot b_{ki} + a_{ii} + \cdot b_{ii} + \sum_{k=i-1}^{n} a_{ik} \cdot b_{ki}$$

=⟨0,0,0⟩

$= a_{ii}.\, b_{ii}$

since a_{ik} = ⟨0,0,0⟩for k = 1,2 …………….. i +1

And b_{ki} =⟨0,0,0⟩ for k = i+1, i+2 ……….. n.

3.1 Determinant of Triangular Fuzzy Matrix

The triangular fuzzy determinant (TFD) of a TFM minor and cofactor are defined as in classical matrices. But, TFD has some special properties due to the sub- distributive property of TFNs.

The triangular fuzzy determinants of TFM A of order nxn is denoted by |A|.

(or) det (A) and is defined as,

$$|A| = \sum_{\sigma \in s_n} S_{gn}\,\sigma\, \langle m_{1\sigma(1)}, \alpha_{1\sigma(1)}, \beta_{1\sigma(1)}\rangle \ldots \langle m_{n\sigma(n)}\rangle$$

$$= \sum_{\sigma \in s_n} S_{gn}\sigma \prod_{i=1}^{n} a_{i_\sigma(1)}$$

where $a_i\sigma_{(i)} = \langle m_i\sigma_{i,\alpha_i}\sigma_i\beta_i\sigma_i\rangle$are TFNS and Sn denotes the symmetric group of all permutations of the indices {1,2,…….n} And Sgn σ= 1 or -1 according as the permutation.

$\sigma = \begin{pmatrix} 1 & 2 & \ldots\ldots\ldots\ldots\ldots & v \\ \sigma\,1 & \sigma\,2 & \ldots\ldots\ldots\sigma & n \end{pmatrix}$ is even or add respectively.

The computation of det (A) involves several product of TFNs. since the product of two or more TFNS is an appropriate TFN the Values of det (A) is also approximate TFN.

Theorem 3.2:

If A is a square TFM then |A|= |A′|

Proof:

Let A = $(a_{ij})_{nxn}$ be a square TFM and A′ = B= $(b_{ij})_{nxn}$ then

$$|B| = \sum_{\sigma \in sn} S_{gn}\sigma b_1\sigma_{(1)}, b_2\sigma_{(2)} \ldots\ldots\ldots b_n\sigma_{(n)}$$

$$= \sum_{\sigma \epsilon sn} S_{gn}\, \sigma a_1\, \sigma_{(1)}, a_2\sigma_{(2)} \ldots\ldots a_n\sigma_{(n)}$$

Let ϕ be the permutation of {1,2,…n} such that $\phi\sigma$ = I, the identity permutation.

Then $\phi = \sigma^{-1}$. Since σ runs over the whole set of permutations, ϕ also runs over the same set of permutations.

Let $\sigma(i)$ = j then $i=\sigma^{-1}(j)$ and $a\sigma(i)i = a_{j\Phi(j)}$. for all I,j. Therefore,

$$|B| = \sum_{\sigma \in sn} Sgn\sigma \left\langle m_{\sigma(1)1}, \alpha_{\sigma(1)1}, \beta_{\sigma(1)1} \right\rangle$$

$$\left\langle m_{\sigma(2)2}, \alpha_{\sigma(2)2}, \beta_{\sigma(1)1} \right\rangle \ldots\ldots \left\langle m_{\sigma(n)n}, \alpha_{\sigma(n)n}, \beta_{\sigma(n)n} \right\rangle$$

$$= \sum_{\phi \in Sn} Sgn\phi \left\langle m_{1\phi(1)}, \alpha_{1\phi(1)}, \beta_{1\phi} \right\rangle$$

$$\left\langle m_{2\phi(2)}, \alpha_{2\phi(2)}, \beta_{2\phi(2)} \right\rangle \ldots\ldots\ldots\ldots\ldots$$

$$= \left\langle m_{n\varnothing(n)}, \alpha_{n\varnothing(n)}, \beta_{n\varnothing(n)} \right\rangle$$

=|A|

Hence |A|=|A′|

4. CIRCULANT TRIANGULAR FUZZY NUMBER MATRICES:

A TFNM $\tilde{A}$ Is said to be circulant TFNM if all the elements of $\tilde{A}$ can be determined completely by its first row.

Suppose the first row of $\tilde{A}$ is

$$\left(\left\langle a_1^1, a_1, a_1^u \right\rangle, \left\langle a_2^1, a_2, a_1^u \right\rangle, \left\langle a_3^1, a_3, a_3^u \right\rangle \ldots, \left\langle a_n^1, a_n, a_n^u \right\rangle \right)$$

then any element a_{ji} of $\tilde{A}$ can be determined (throughout the element of the first row) as

$$A_{ij} = a_{1\langle n \oplus i+j+1 \rangle} \text{ with } a_{1(n+k)} = a_{1k}$$

A circulant TFNM is the form of

$$\begin{pmatrix} \langle a_1^1, a_1, a_1^u \rangle & \langle a_2^1, a_2, a_2^u \rangle & \cdots & \langle a^1_{n\oplus 1}, a_{n\oplus 1}, a^u_{n\oplus 1} \rangle & \langle a_n^1, a_n, a_n^u \rangle \\ \langle a_n^1, a_n, a_n^u \rangle & \langle a_n^1, a_1, a_1^u \rangle & \cdots & \langle a^1_{n\oplus 2}, a_{n\oplus 2}, a^u_{n\oplus 2} \rangle & \langle a^1_{n\oplus 1}, a_{n\oplus 1}, a^u_{n\oplus 1} \rangle \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \langle a_3^1, a_3, a_3^u \rangle & \langle a_4^1, a_4, a_4^u \rangle & \cdots & \langle a_1^1, a_1, a_1^u \rangle & \langle a_2^1, a_2, a_2 \rangle \\ \langle a_2^1, a_2, a_2^u \rangle & \langle a_3^1, a_3, a_3^u \rangle & \cdots & \langle a_n^1, a_n, a_n^u \rangle & \langle a_1^1, a_1, a_1^u \rangle \end{pmatrix}$$

Theorem 4.1:

An nxn TFNM $\tilde{A}$ is circulant if and only if $\tilde{A}\tilde{c}_n = \tilde{C}_n\tilde{A}$,

Where, $\tilde{C}_n$ is the permutation matrix of unit TFNM.

$$C_n = \begin{pmatrix} \langle 0,0,0\rangle & \langle 0,0,0\rangle & \cdots & \langle 0,0,0\rangle & \langle 0,1,0\rangle \\ \langle 0,1,0\rangle & \langle 0,0,0\rangle & \cdots & \langle 0,0,0\rangle & \langle 0,0,0\rangle \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \langle 0,0,0\rangle & \langle 0,0,0\rangle & \cdots & \langle 0,0,0\rangle & \langle 0,0,0\rangle \\ \langle 0,0,0\rangle & \langle 0,0,0\rangle & \cdots & \langle 0,1,0\rangle & \langle 0,0,0\rangle \end{pmatrix}$$

Proof:

Let $\tilde{A}$ be a TFNM and $\tilde{P} = \tilde{A}\tilde{C}_n$, then $p_{ij} = \sum_{k=1}^{n} \langle a_{ik} C_{kj} \rangle$ Since, only c_{1n} is ⟨0,0,0⟩.$p_{ij} = a_{1(j\oplus 1)}$.

Similarly, if $\tilde{T} = \tilde{C}_n\tilde{A}$, then $t_{ij} = a_{(1\oplus \overline{n-1})j}$. so, by Note 4.1.3 $p_{ij} = t_{ij}$ for all I, j $\in$n.

Hence $\tilde{A}\tilde{C}_n = \tilde{C}_n\tilde{A}$.

So, $\tilde{A}$ is circulant TFNM.

Converse is straight forward.

Let $\tilde{A}$ and $\tilde{c}$ be two circulant TFNMs of order 3x3, where

$$\tilde{A} = \begin{pmatrix} \langle 2,3,4\rangle & \langle 4,6,7\rangle & \langle 3,6,7\rangle \\ \langle 3,6,7\rangle & \langle 2,3,4\rangle & \langle 4,6,7\rangle \\ \langle 4,6,7\rangle & \langle 3,6,7\rangle & \langle 2,3,4\rangle \end{pmatrix} \text{ and } \tilde{C} = \begin{pmatrix} \langle 0,0,0\rangle & \langle 0,0,0\rangle & \langle 0,1,0\rangle \\ \langle 0,1,0\rangle & \langle 0,0,0\rangle & \langle 0,0,0\rangle \\ \langle 0,0,0\rangle & \langle 0,1,0\rangle & \langle 0,0,0\rangle \end{pmatrix}$$

Then

$$\tilde{A}\tilde{C} = \begin{pmatrix} \langle 4,6,7\rangle & \langle 3,6,7\rangle & \langle 2,3,4\rangle \\ \langle 2,3,4\rangle & \langle 4,6,7\rangle & \langle 3,6,7\rangle \\ \langle 3,6,7\rangle & \langle 2,3,4\rangle & \langle 4,6,7\rangle \end{pmatrix} \text{ and } \tilde{C}\tilde{A} = \begin{pmatrix} \langle 4,6,7\rangle & \langle 3,6,7\rangle & \langle 2,3,4\rangle \\ \langle 2,3,4\rangle & \langle 4,6,7\rangle & \langle 3,6,7\rangle \\ \langle 3,6,7\rangle & \langle 2,3,4\rangle & \langle 4,6,7\rangle \end{pmatrix}$$

Thus $\tilde{A}\tilde{C} = \tilde{C}\tilde{A}$.

Theorem 4.2:

For the circulant TFNMS $\tilde{A}$and $\tilde{B}$.

i) $\tilde{A} + \tilde{B}$ is a circulant TFNM.

ii) $\tilde{A}'$is a circulant TFNM.

iii) $\tilde{A}\tilde{B}$is also a circulant TFNM. In particular, $\tilde{A}^k$ is also a circulant TFNM.

iv) $\tilde{A}\tilde{A}'$is circulant TFNM.

Proof:

(i) Proof is Straightforward.

(ii) Since $\tilde{A}$ is circulant TFNM then $\tilde{A}$ commutes with $\tilde{C}_n$.

So, $\tilde{A}\tilde{C}_n = \tilde{C}_n\tilde{A}$. Transposing both sides of $\tilde{A}\tilde{C}_n = \tilde{C}_n\tilde{A}$, $\tilde{C}_n$, $\tilde{A}= \tilde{A}$, $\tilde{C}_n'$

or, $\tilde{C}_n\tilde{C}_n'\tilde{A}' = \tilde{C}_n\tilde{A}'.\tilde{C}_n'$

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or, $\tilde{A}' = \tilde{C}_n \tilde{A}' \tilde{C}'_n$ $since, \tilde{C}_n . \tilde{C}_n = I = \tilde{C}_n \tilde{A} . A$

or, $\tilde{A} . \tilde{C}_n . = \tilde{C}_n \tilde{A} . \tilde{C}_n . \tilde{C}_n = \tilde{C}_n \tilde{A} .$

so, $\tilde{A} .$ *is circulant TFNS*

iii) Since, $\widetilde{A}$ and $\widetilde{B}$ are circulant TFNMs, each $\widetilde{A}$ and $\widetilde{B}$ commutes with $\tilde{C}_n$.

Hence $\widetilde{A}\widetilde{B}$ commutes with $\tilde{C}_n$.

So, by for a circulant TFNM $\widetilde{A}$notice that $a_{(i\oplus \overline{n-1})} = a_i$ $(j\oplus 1)$

for every I,j $\in \{1,2,\ldots\ldots,n\}$. Theorem 4.1.1,$\tilde{A}\tilde{B}$ is circulant TFNM. $\tilde{A}\tilde{A}'$is circulant TFNM.

CONCLUSION

In this project some elementary operations on triangular fuzzy numbers are defined, like Classical matrices. Also define some operations on Triangular Fuzzy Matrices using the elementary operations. Some important properties of Triangular Fuzzy Matrices are presented. The concept of adjoint of Triangular Fuzzy Matrices is discussed and some properties on it are also presented. The definition and some properties of determinant of Triangular Fuzzy Matrix are presented in this project. It is well known that the determinant is a very important tool in mathematics so an efficient method is required to evaluate a Traingular Fuzzy Determinant Presently. The research are trying to develop an efficient method to evaluate a Triangular Fuzzy Determinant of large size, some special types of Triangular Fuzzy Matrices. The concept of adjoint of circulant TFNM is discussed and some properties of determinant of circulant TFNM are presented in this article. Finally define generalized TFNM and also investigate the distance measure of generalized circulant TFNMs.

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